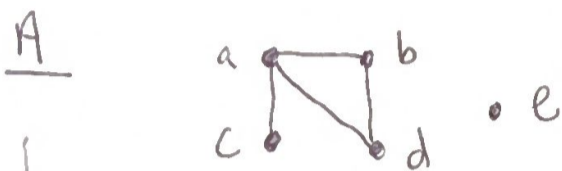


17 - Graphs

A (finite, simple) graph $G = (V, E)$ consists of a nonempty finite set V and a set E of 2-element subsets of V . Elements of V & E are called vertices and edges, respectively. We often abbreviate an edge $\{u, v\}$ to uv .

Ex 1 Draw the graph $G = (V, E)$ with vertex set $V = \{a, b, c, d, e\}$ and edge set $E = \{ab, ac, ad, bd\}$.



Def'n Let $G = (V, E)$ be a graph. Vertices u, v are adjacent if $\{u, v\} \in E$. The degree of vertex v is

$$\deg(v) \stackrel{\text{def'n}}{=} |\{u \in V : u \text{ is adjacent to } v\}|.$$

Ex 2 In above graph:

vertex	a	b	c	d	e
degree	3	2	1	2	0

Notice each edge has 2 endpoints, so each edge contributes +2 to the sum of degrees:

Handshaking Lemma The sum of degrees of all vertices of a graph is twice the number of edges:

$$\sum_{v \in V} \deg(v) = 2 \cdot |E|.$$

or $|E| = \frac{1}{2} \sum_{v \in V} \deg(v)$